A2: Quadratics, cubics and quartics

Question 23

$x^3 + 2x^2 + ax - 6$ is exactly divisible by $x - 2$. Find the value of a.

[Adapted from VCAA 1999 MM]

Question 24

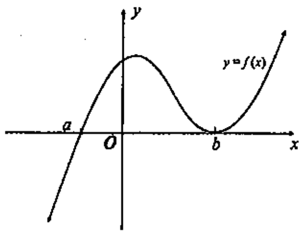
At $(-1, -4)$, the graph of the function f, with rule $f(x) = (x+1)^3 - 4$ has

A. an x-axis intercept.
B. its y-axis intercept.
C. a local minimum.
D. a local maximum.
E. point of inflection with zero gradient.

[Adapted from VCAA 1999 MM]

Question 25

The graph shown here could be that of a function f whose rule is



A. $f(x) = (x-a)^2(x-b)$
B. $f(x) = (x+a)^2(x-b)$
C. $f(x) = (x-a)^2(x-b)$
D. $f(x) = (x-a)^2(x+b)$
E. $f(x) = (x+a)(x+b)^2$

[Adapted from VCAA 2000 MM]

Question 26

Let $f: R \to R$, $f(x) = 3x^2 - 12x + 18$

a. Write $f(x)$ in the form $a(x+b)^2 + c$.

b. Hence, write down the coordinates of the turning point of the graph of f.

[Adapted from VCAA 2001 MM]

Question 27

The linear factors of $x^4 + x^3 - 4x^2 - 4x$ over R are

A. $x, x+1, x^2+4$
B. $x, x+1, x+2, x-2$
C. $x, x+1$
D. $x+1, x+2, x-2$
E. $x+1, x^2+4x$

[Adapted from VCAA 2002 MM]

Question 28

Use the remainder theorem to determine if $3x^4 + 7x^2 + 7x + 11$ is exactly divisible by $(x-1)$.

[Adapted from VCAA 2003 MM]

Question 29

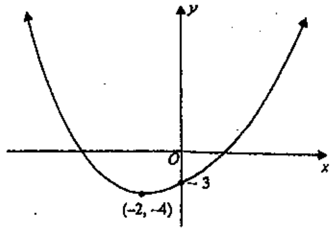
Given that $f(x) = 2[(x+2)^2 - 3]$, the coordinates of the turning point of the graph of f are

A. $(-2, -3)$ B. $(4, -6)$ C. $(-2, -6)$
D. $(2, -3)$ E. $(-4, -6)$

[Adapted from VCAA 2003 MM]

Question 30

The following shows part of the graph of the curve with equation $y = A(x+B)^2 - 4$. The values of A and B respectively could be

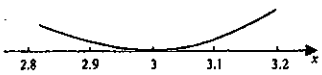


	A	B
A.	$\frac{1}{4}$	-4
B.	4	2
C.	-2	4
D.	$-\frac{1}{4}$	2
E.	$\frac{1}{4}$	2

[Adapted from VCAA 2004 MM]

Question 31

Which one of the following could be the rule for the cubic equation, part of which is shown here. The turning point at (3, 0) is shown.



A. $f(x) = x(x+3)^2$
B. $f(x) = x(x-3)^2$
C. $f(x) = (x+3)^2$
D. $f(x) = x^2(x-3)$
E. $f(x) = -x(x-3)^2$

[Adapted from VCAA 2003 MM]

Question 32

Let f be a cubic polynomial. The graph of $y = f(x)$ either intersects or touches the x-axis at exactly two points $(a, 0)$ and $(b, 0)$. A possible rule for f could be

A. $f(x) = (x-a)(x-b)$
B. $f(x) = (x-a)(x+b)^2$
C. $f(x) = (x-a)(x-b)^2$
D. $f(x) = (x+a)^2(x-b)$
E. $f(x) = (x+a)^2(x+b)$

[Adapted from VCAA 2004 MM]

Question 33

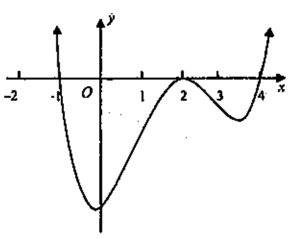
Which one of the following is not a factor of $x^4 + 3x^3 - 4x^2 - 12x$?

A. $x-2$ B. $x+2$ C. x D. $x-3$ E. $x+3$

[Adapted from VCAA 2005 MM]

Question 34

Part of the graph of the function f is shown here. The rule for f is most likely to be



A. $f(x) = (x+2)^2(x-1)(x-4)$
B. $f(x) = (x-2)(x+1)^2(x+4)$
C. $f(x) = (2-x)(x+1)^2(x+4)$
D. $f(x) = (x-2)^2(x+1)(x-4)$
E. $f(x) = (x+2)(x-1)^2(4-x)$

[Adapted from VCAA 2005 MM]

A2: Quadratics, cubics and quartics

Question 35

If $f(x) = -2x^2 - 2x + 1$ then $f(-2)$ is equal to

A. -11 B. 3 C. -3 D. 13 E. 11

Question 36

If $f(x) = 3x^3 - 2x - 1$ then $f(-1)$ is equal to

A. 0 B. -4 C. 4 D. 2 E. -2

Question 37

a. Find the coordinates of the points of intersection of the curve with equation $y = x^2 - 2x - 1$ and the line with equation $y = 3x - 5$.

~~b. Find the distance between the two points found in part a.~~

Question 38

a. Find the equation of the cubic, $f(x)$, that just touches the x-axis at $x = -2$, contains the point (1, 18) and passes through the origin.

b. Find the equation of the cubic, $g(x)$, that is obtained by shifting $f(x)$ from part a two units in the positive direction of the x-axis.

Question 39

'Completing the square' on the equation $x^2 + 7x + 1$ gives

A. $(x+7)^2 - 11\frac{1}{4}$ B. $(x-7)^2 - \frac{45}{4}$ C. $(x+\frac{7}{2})^2 - 1$
D. $(x+\frac{49}{4})^2 - \frac{1}{4}$ E. $(x+\frac{7}{2})^2 - \frac{45}{4}$

Question 40

The coordinates of the turning point of the graph of $y = -2x^2 - 28x - 1$ are

A. $(-7, 97)$ B. $(-7, -295)$ C. $(7, -785)$ D. -7 E. 7

Question 41

The range of k values over which the quadratic with equation $y = x^2 - 6x + k$ will have two separate solutions is

A. $k < 9$ B. $k \leq 9$ C. $k = 9$ D. $k > 9$ E. $k \geq 9$

Question 42

The quadratic expression $(x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$ is equivalent to

A. $x^2 - 2x + 5$ B. $x^2 + 2x - 5$ C. $x^2 + 2\sqrt{5}x - 1$
D. $x^2 - 2x - 4$ E. $x^2 + 2x + 4$

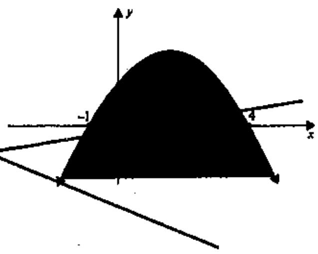
Question 43

$(x + y)^2 - (x - y)^2$ is equivalent to

A. 0 B. $4xy$ C. x^2 D. y^2 E. $x^2 - y^2$

~~**Question 44**~~

~~The inequation describing the shaded area of the graph shown here is~~



A. $y \geq -x^2 + 3x + 4$
B. $y \leq -x^2 + 3x + 4$
C. $y \leq x^2 + 3x - 4$
D. $y \geq x^2 + 3x - 4$
~~E. $y \geq x^2 + x - 4$~~

Question 45

The solution set to the quadratic inequality $x^2 + 6x + 9 \leq 0$ is

A. $\{-3\}$ B. $\{x: 6 \leq x \leq 9\}$ C. $\{x: x \leq 6\} \cup \{x: x \geq 9\}$
D. R E. $R \setminus \{-3\}$

Question 46

The expansion of $(2a - b)^3$ is

A. $4a^3 - 2a^2b + ab^2 - b^3$ B. $8a^3 - 12a^2b + 6ab^2 - b^3$
C. $8a^3 + 12a^2b + 6ab^2 + b^3$ D. $8a^3 - b^3$
E. $(2a - b)(4a^2 + 2ab + b^2)$

Question 47

The solutions to the equation $(3 - x)(2x - 3)(4 - 3x) = 0$ are

A. $-3, \frac{3}{2}, \frac{4}{3}$ B. $3, \frac{3}{2}, \frac{4}{3}$ C. $3, \frac{2}{3}, \frac{3}{4}$ D. $-3, \frac{2}{3}, \frac{3}{4}$ E. 3 and 4

Question 48

The cubic $x^3 - kx + 2$ is exactly divisible by $x + 2$. The value of k must be

A. 3 B. −3 C. 5 D. −5 E. 2

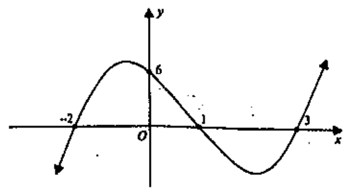
Question 49

Expressed in factorized form, the cubic $x^3 + 2x^2 - x - 2$ becomes

A. $(x - 1)(x + 1)(x + 2)$ B. $(x - 1)(x + 1)(x - 2)$
C. $(x - 1)(x + 2)(x - 3)$ D. $(x - \sqrt{2})(x + \sqrt{2})(x - 1)$
E. $(x + 1)(x - 2)(x - \frac{1}{3})$

Question 50

The equation best matching the graph shown is

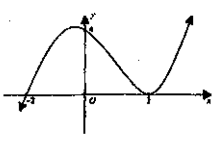


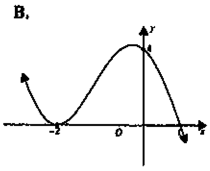
A. $y = (x + 1)(x - 2)(x + 3)$
B. $y = (x - 1)(x + 2)(x - 3)$
C. $y = (x + 6)(x + 1)(x - 2)$
D. $y = (x - 1)(x - 2)(x - 3)$
E. $y = (x + 1)(x + 2)(x - 3)$

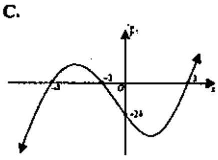
A2: Quadratics, cubics and quartics

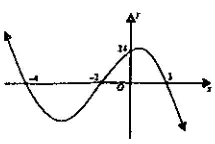
Question 51

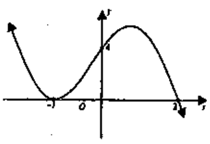
Which of the following graphs is best represented by the equation $y = (x + 2)^2(1 - x)$?

A.


B.


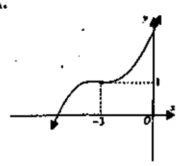
C.


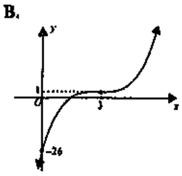
D.


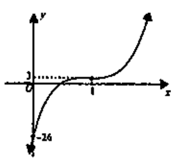
E.


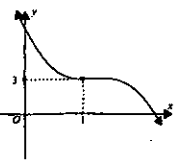
Question 52

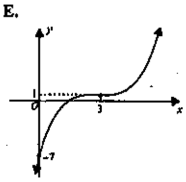
Which of the following graphs is best represented by the equation $y = (x - 3)^3 + 1$?

A.


B.


C.


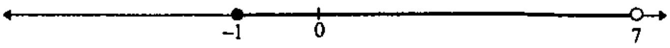
D.


E.


A3:Domain, range and functions

A3: Domain, range and functions

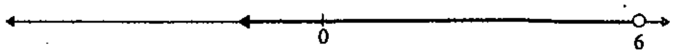
Question 53



This number-line section is best described by the inequality

A. (-1, 7] B. [-1, 7) C. (-1, 7) D. [-1, 7] E. [7, -1)

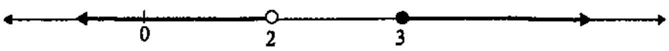
Question 54



This number-line section is best described by the set

A. $(6, -\infty)$ B. $(-\infty, 6]$ C. $[-\infty, 6]$ D. $(-\infty, 6)$ E. $[-\infty, 6)$

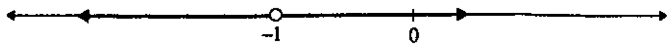
Question 55



This number-line section is best described by the set

A. $\{x: x < 2\} \cup \{x: x \geq 3\}$ B. $\{x: x \leq 2\} \cup \{x: x > 3\}$ C. $\{x: x < 2\} \cup \{x: x > 3\}$
D. $\{x: x \leq 2\} \cap \{x: x > 3\}$ E. $\{x: x \leq 3\} \cup \{x: x > 2\}$

Question 56



The numbers shown are described by the set

A. $\{-1\}$ B. $R \setminus -1$ C. $R \setminus \{-1\}$ D. $R \cup \{-1\}$ E. $R \cap \{-1\}$

Question 57

The set of numbers $\{x: x \geq 0\}$ is equivalent to the set

A. $R^+ \cap \{0\}$ B. $R^+ \cup \{0\}$ C. $R \cup \{0\}$ D. R^+ E. $R^+ \setminus \{0\}$

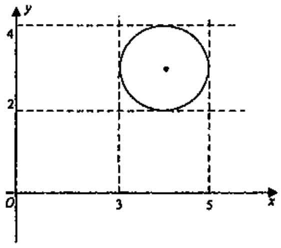
Question 58

The set of numbers $R \setminus \{3\}$ is equivalent to the set

A. $\{x: x < 3\} \cup \{x: x > 3\}$ **B.** $\{x: x \le 3\} \cup \{x: x \ge 3\}$ **C.** $\{x: x < 3\} \cup \{x: x \ge 3\}$
D. $\{x: x < 3\} \cap \{x: x > 3\}$ **E.** $\{x: 3 > x < 3\}$

Question 59

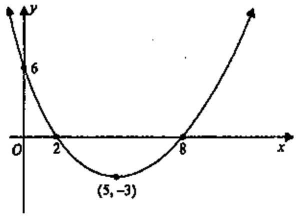
The domain of this graph is



A. $\{y: 2 \le y \le 4\}$ **B.** $\{x: 3 < x < 5\}$
C. $\{y: 2 < y < 4\}$ **D.** $\{x: 3 \le x \le 5\}$
E. $\{x: 3 \le x \le 5\} \cup \{y: 2 \le y \le 4\}$

Question 60

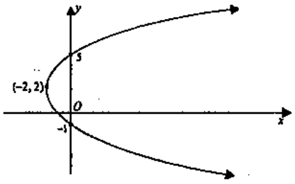
The range of this graph is equal to



A. $\{y: y \ge -3\}$
B. $\{y: y \ge 5\}$
C. $\{x: 2 \le x \le 8\}$
D. $\{y: y \ge 6\}$
E. $\{x: x \le 2\} \cup \{x: x \ge 8\}$

Question 61

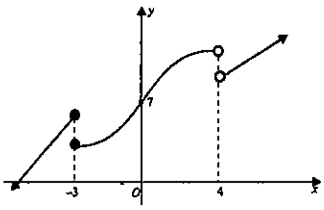
The domain of this graph is equal to



A. $\{y: -1 \le y \le 5\}$
B. $\{y: y \ge 5\}$
C. $\{x: x \ge -1\}$
D. $\{x: x \le -1\}$
E. $\{x: x \ge -2\}$

Question 62

The domain of this graph is equal to



A. $R \setminus \{7\}$
B. $R^+ \cup \{7\}$
C. $R^+ \cup \{4\}$
D. $R \setminus \{4\}$
E. $R \setminus \{-3, 4\}$

Question 63

The largest possible (maximal) domain of the graph with equation $y = \sqrt{x-1}$ is equal to

A. $\{x: x \ge 1\}$ **B.** $\{x: x \le 1\}$ **C.** $\{x: x > 1\}$
D. $\{x: x \ge 0\}$ **E.** $\{x: x \ge -1\}$

Question 64

The maximal domain of the graph with equation $y = \frac{2}{3x-2}$ is equal to

A. $R \setminus \{3\}$ **B.** $R \setminus \{2\}$ **C.** $\{x: x \ge 2\}$
D. $R \setminus \{-\frac{2}{3}\}$ **E.** $R \setminus \{\frac{2}{3}\}$

Question 65

The range of the graph with equation $y = \sqrt{x-1} - 3$ is equal to

A. $\{y: y \ge -3\}$ **B.** $\{x: x \ge -1\}$ **C.** $\{x: x \le -3\}$
D. $\{y: y \ge 1\}$ **E.** $\{y: y \ge 0\}$

Question 66

The domain and range respectively for the function with rule $y = 2(x-3)^2 - 1$ are

A. $(3, 1), R$ **B.** $R, \{y: y \ge 3\}$ **C.** $R, \{y: y \ge -1\}$
D. $\{x: x \ge 3\}, \{y: y \ge -1\}$ **E.** $\{x: x \ge 6\}, \{y: y \ge 1\}$

Question 67

The relation $y^2 = (3 - x)$ is not a function. This is because

A. for some x values in the domain, there are two y values
B. x cannot be greater than 3
C. there is only an upper half of a graph
D. the range is positive only
E. the power of ½ and the surd square root are different.

A3: Domain, range and functions

Question 68

The equation of a circle of radius 9 and centre (3, 2) is $(x-3)^2 + (y-2)^2 = 81$. It is defined over the maximal domain. This is not a function because

A. the domain and range are both the same.
B. the centre is $\sqrt{13}$ units from the origin.
C. there are no points on the graph more than 9 units from its centre.
D. the domain and range are not stated fully.
E. there is a point on its graph with the same x-coordinate as another point on the graph.

~~**Question 69**~~

~~If $f(x) = -|x|$, then the range of $f(x)$ is~~

~~**A.** $\{y: y \le 0\}$ **B.** $\{y: y < 0\}$ **C.** $\{y: y > 0\}$ **D.** $\{y: y \ge 0\}$ **E.** 0~~

Question 70

If $f(x) = 2x^2 + 3$ then $f(-x)$ is equal to

A. $f(x)$ **B.** $-f(x)$ **C.** 5 **D.** −5 **E.** 3

Question 71

If $f(x) = \begin{cases} x^2 & x < 0 \\ 2x & x = 0 \\ 2x^3 & x > 0 \end{cases}$ then $f(-1)$ is equal to

A. 1 **B.** −1 **C.** 2 **D.** −2 **E.** 3

~~**Question 72**~~

~~The inverse of the function $y = 2x - 3$ is~~

A. $y = 2 - 3x$ **B.** $y = \frac{1}{2x-3}$ **C.** $y = \frac{1}{2}(x+3)$
D. $x = \frac{1}{2}(y+3)$ **E.** $y = \frac{1}{2}x - \frac{1}{3}$

A3: Domain, range and functions

~~**Question 73**~~

~~The inverse rule for the function $y = x^2 - 1$ is~~

A. $y = \sqrt{x+1}$ **B.** $y = \sqrt{x-1}$ **C.** $y = \pm\sqrt{x+1}$
D. $y = -\sqrt{x+1}$ **E.** $y = \pm\sqrt{x-1}$

~~**Question 74**~~

The domain of the inverse of the function $y = x^2 + 1$ is

A. $\{x: x \le 1\}$ **B.** $R^+ \cup \{0\}$ **C.** R **D.** $\{x: x \ge 1\}$ **E.** $\{x: x > 1\}$

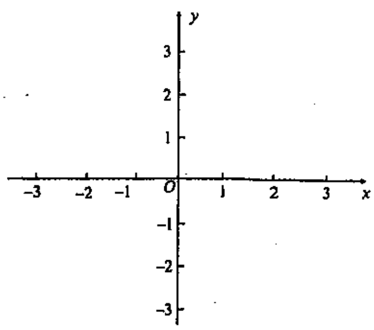
Question 75

Let $f(x) = \frac{x}{x+1}$ where D is the largest subset of R for which f is defined.

a. By using long division or CAS techniques (which should be explained), express $f(x)$ in the form $f(x) = \frac{a}{x+b} + c$.

b. State the domain of $f(x)$.

c. On the axes given here, sketch the graph of the function with equation $y = f(x)$. Clearly mark the coordinates of the points of intersection with the axes. Clearly label any asymptotes with their equations.



[Adapted from VCAA 2001 MM]

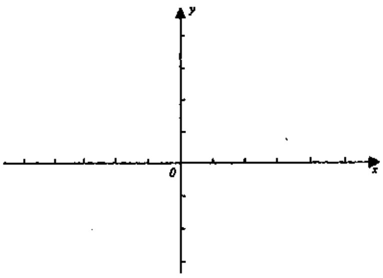
Question 76

A function f is defined by the rule $f(x) = 4 - 2^{-x}$.

a. State the exact value of $f(0)$.

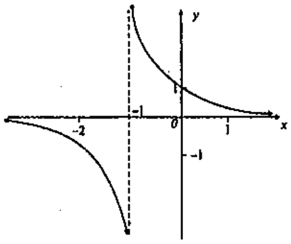
b. Find the x-intercept/s of the graph of $f(x)$, giving your answer with exact values.

c. Sketch the graph of f on the axes given here, clearly labelling any axial intercepts with their coordinates, and clearly labelling any asymptotes with their equations.



Question 77

The following is the graph of a function with equation $y = \frac{a}{(x+b)}$. The values of a and b respectively are



	a	b
A.	−1	1
B.	−1	−1
C.	1	−1
D.	1	1
E.	−2	2

[Adapted from VCAA 1999 MM]

Question 78

Which one of the following is not true of the graph of the function f with rule $f(x) = 2^x$?

A. It has range R^+.
B. It has domain R.
C. It has a horizontal asymptote at $y = 0$.
D. It passes through the point (0, 2).
E. The slope of the tangent at any point on the graph is positive.

[VCAA 2004 MM]

Question 79

The equations of the vertical and horizontal asymptotes of the graph whose equation is $y = \frac{2}{x-4} + 3$ are respectively

A. $x = -4,\ y = 3$ B. $x = 2,\ y = 3$ C. $x = 3,\ y = 4$
D. $x = 4,\ y = -3$ E. $x = 4,\ y = 3$

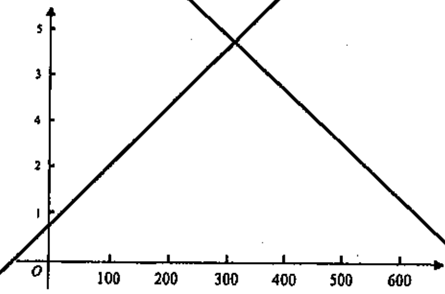
[VCAA 2000 MM]

Question 80

A well-designed computer screen display aims to make it quick and easy to do tasks such as clicking an icon. Duncan's Law states that the time taken to move to a point and click it depends on the distance the mouse is moved and the width of the actual screen.

According to Duncan's Law, the time taken to move to a point, in seconds, is given by $f(x) = a - b\log_{10}(x), 0 < x \le 640$, where x pixels is the screen width and a and b are positive constants for a particular user. Annie discovers that, for her, $a = 2.5$ and $b = 0.9$. Let $f: (0, 600] \to R,\ f(x) = 2.5 - 0.9\log_{10}(x)$.

a. Using graphical calculator techniques or otherwise, sketch the graph of $y = f(x)$ on the axes shown below. Label the axes and label any asymptote with its equation.



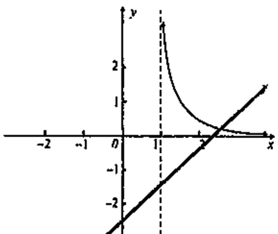
b. Find the screen width, correct to the nearest pixel, for which Annie will take 2 seconds to move to a point.

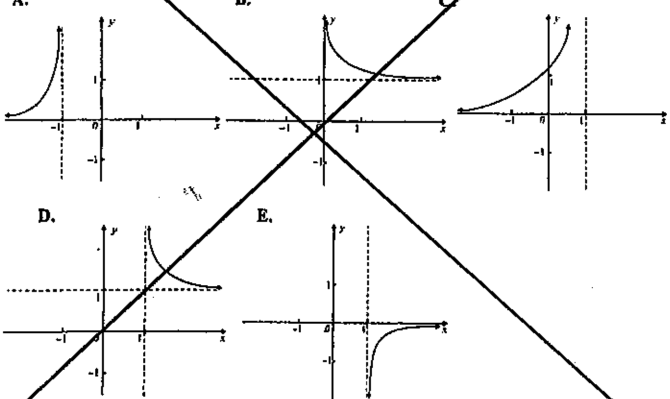
[Adapted from VCAA 2004 MM]

A4: Functions and transformations

Question 81

The graph of the function with equation $y = f(x)$ is shown. Which one of the following is most likely to be the graph of $y = f^{-1}(x)$, the inverse function?





[VCAA 1999 MM]

Question 82

The graph, whose equation is $y = \sqrt{x}$, is reflected in the x-axis and then translated 2 units to the right and 1 unit down. The equation of the new graph is

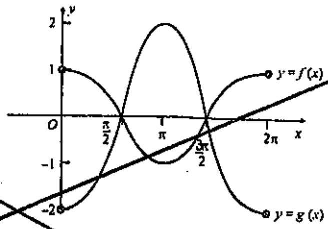
A. $y = \sqrt{(x-2)} + 1$ B. $y = -\sqrt{(x-2)} - 1$ C. $y = -\sqrt{(x+2)} - 1$
D. $y = -\sqrt{(x-2)} + 1$ E. $y = \sqrt{(x-1)} + 2$

[VCAA 2000 MM]

A4: Functions and transformations

Question 83

The diagram shows the graphs of two circular functions, f and g.



The graph of the function with equation $y = f(x)$ is transformed into the graph of the function with equation $y = g(x)$ by

A. a dilation by a scale factor of ½ from the y-axis and a reflection in the x-axis.
B. a dilation by a scale factor of ½ from the x-axis and a reflection in the x-axis.
C. a dilation by a scale factor of 2 from the x-axis and a reflection in the x-axis.
D. a dilation by a scale factor of 2 from the y-axis and a reflection in the y-axis.
E. a dilation by a scale factor of 2 from the x-axis and a reflection in the y-axis.

[VCAA 2001 MM]

Question 84

The graph of the function with rule $y = \frac{1}{x}$ is transformed as follows:

a dilation by a factor of 2 from the y-axis, a reflection in the y-axis, a translation of 1 unit parallel to the x-axis, a translation of +1 unit parallel to the y-axis.

a. Write down the equation of the rule of the transformed function.

b. Hence state the domain and range of the transformed function.

[Adapted from VCAA 2002 MM]

Question 85

The graph of the function f is obtained from the graph of the function with equation $y = \sqrt{x}$ by a reflection in the y-axis followed by a dilation of 3 units from the x-axis. The rule for f is

A. $f(x) = -3\sqrt{x}$ B. $f(x) = \sqrt{-3x}$ C. $f(x) = \sqrt{-\frac{1}{3}x}$
D. $f(x) = 3\sqrt{x}$ E. $f(x) = 3\sqrt{-x}$

[Adapted from VCAA 2003 MM]

Question 86

a. The graph of g is obtained from the graph of the function f with rule $f(x)=x^2$ by a translation by −4 units parallel to the x-axis. Write down the rule for g.

b. The graph of h is obtained from the graph of g by a translation by +3 units parallel to the y-axis. Write down the rule for h.

c. The graph of k is obtained from the graph of h by a dilation by a scale factor of 0.5 from the y-axis. Write down the rule for k.

[Adapted from VCAA 2004 MM]

Question 87

The graph of the function with rule $y=x^3$ is transformed as follows

- a translation of −1units parallel to the x-axis and then
- a dilation by a factor of $\frac{1}{3}$ from the x-axis

The rule of the function corresponding to the transformed graph is

A. $f(x)=\frac{1}{3}(x-1)^3$ B. $f(x)=3(x-1)^3$ C. $f(x)=\left(\frac{x}{3}-1\right)^3$

D. $f(x)=\frac{1}{3}(x+1)^3$ E. $f(x)=(3x+1)^3$

~~**Question 88**~~

The graph of the function f with rule $f(x)=\sin(x)$ is transformed to the graph of the function g with rule $g(x)=5\sin(3x)$ by

A. a dilation from the x-axis by a scale factor of 5 and a dilation from the y-axis by a scale factor of 3.

B. a dilation from the x-axis by a scale factor of 3 and a dilation from the y-axis by a scale factor of 5.

C. a dilation from the x-axis by a scale factor of $\frac{1}{3}$ and a dilation from the y-axis by a scale factor of 5.

D. a dilation from the x-axis by a scale factor of $\frac{1}{5}$ and a dilation from the y-axis by a scale factor of 3.

E. a dilation from the x-axis by a scale factor of 5 and a dilation from the y-axis by a scale factor of $\frac{1}{3}$.

[Adapted from VCAA 2004 MM]

~~**Question 89**~~

The graph of the curve with rule $y=f(x)$, where f is a one-to-one function, has exactly one asymptote whose equation is $x=3$. The graph of the curve with rule $y=f^{-1}(x)$, where f^{-1} is the inverse function of f, will have

A. a horizontal asymptote at $y=-3$.

B. a horizontal asymptote at $y=\frac{1}{3}$.

C. a horizontal asymptote at $y=3$.

D. a vertical asymptote at $x=\frac{1}{3}$.

E. a vertical asymptote at $x=3$.

[Adapted from VCAA 2004 MM]

Question 90

The graph of the function with rule $y=x^2$ is transformed and becomes the graph with rule $y=3(x-2)^2$.

Describe the transformations that will produce this change.

A6: Logarithms and indices

A6: Logarithms and indices

Question 112

If $\log_a b=c$ then

A. $a^c=b$ B. $c^a=b$ C. $b^a=c$ D. $a^b=c$ E. $c^b=a$

Question 113

Since it is true that $125=5^3$, then

A. $\log_5 125=3$ B. $\log_5 3=125$ C. $\log_{125} 5=3$
D. $\log_{125} 3=5$ E. $\log_3 125=5$

Question 114

$\log_3 81$ is equal to

A. 4 B. 3 C. 2 D. 1 E. 15

Question 115

The x values for which the function $y=\log_{10}(x-3)$ is defined are

A. $x\ge 0$ B. $x>0$ C. $x>3$ D. $x\ge 3$ E. $x<3$

Question 116

The x values for which the function $y=\log_{10}(5-2x)$ is defined are

A. $x<\frac{2}{5}$ B. $x\ge\frac{5}{2}$ C. $x>\frac{5}{2}$ D. $x<\frac{5}{2}$ E. $x\le\frac{5}{2}$

Question 117

$\log_a 3+\log_a 2$ is equal to

A. $\log_a 5$ B. $\log_a 6$ C. $\log_a 15$ D. $\log_a 9$ E. $\log_a 27$

Question 118

$3\log_x 5$ is equal to

A. $\log_x \frac{5}{3}$ B. $\log_x 2$ C. $\log_x 8$ D. $\log_x 15$ E. $\log_x 125$

Question 119

$\log_3 5 + \log_3 9 - \log_3 15$ simplifies to give

A. $\log_3 -1$ B. $\log_3 30$ C. 1 D. 0 E. $\log_3 \frac{14}{15}$

Question 120

$\log_a (x^3 y^2)$ is equal to

A. $(3x + 2y)\log_a xy$
B. $3x^2 \log_a x + 2y^3 \log_a y$
C. $2\log_a x + 3\log_a y$
D. $x\log_a 3 + y\log_a 2$
E. $3\log_a x + 2\log_a y$

Question 121

$\log_3 6$ can be re-written as

A. $\log_3 2 + \log_2 3$ B. $\log_2 3 + 1$ C. $\log_3 2 + 1$
D. $6\log_3$ E. 2

Question 122

$\log_3 \sqrt{3}$ is equal to

A. $\frac{1}{2}$ B. 2 C. -2 D. $3\sqrt{3}$ E. $\sqrt{3}$

Question 123

$\frac{1}{3}\log_2 64$ simplifies to give

A. 12 B. 192 C. $21\frac{2}{3}$ D. 2 E. 4

Question 124

$\frac{\log_{10} 125}{\log_{10} 25}$ simplifies to give

A. 5 B. $\log_{10} 5$ C. $\log_{10} 1\frac{1}{2}$ D. $1\frac{1}{2}$ E. 2

Question 125

If $x\log_{10} 3 = \frac{3}{2}\log_{10} 9$ then x is equal to

A. 1 B. 2 C. 3 D. 4 E. 5

Question 126

If $\log_{10} x - \log_{10}(x-3) = \log_{10} 4$ then x is equal to

A. 9 B. 7 C. 4.5 D. 12 E. 4

Question 127

Simplifying $2\log_{10} 4 - 2\log_{10} 5 + \log_{10} 50$ gives

A. 16 B. $5\log_{10} 2$ C. 32 D. $4\log_{10} 2$ E. $\log_{10} 40$

Question 128

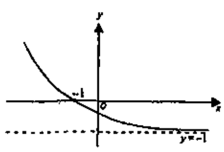
If $2^x = 5$ then x is approximately equal to

A. 0.40 B. 2.4 C. 2.32 D. 3 E. $\frac{5}{2}$

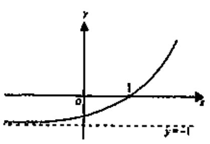
Question 129

The graph of $y = \log_{10}(x-1)$ is best shown by

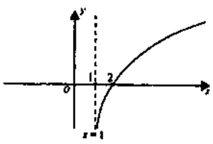
A.



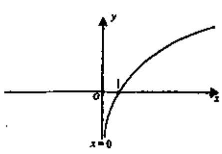
B.



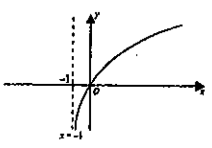
C.



D.



E.



Question 130

If $\left(\frac{1}{2}\right)^x > 4$ then

A. $x > -2$ B. $x \geq -2$ C. $x \geq \frac{1}{2}$ D. $x < -2$ E. $x \leq \frac{1}{2}$

For questions 131 to 133, use this information.
The number of bacteria in a dish is equal to N where $N = 1200(2)^{0.1t}$ and t is the time in hours since the start of an experiment.

Question 131

The initial number of bacteria is

A. 1200 B. 0.1 C. 2 D. $\log_2 1200$ E. 1

Question 132

The number of bacteria after 2 days is

A. 1378 B. 33429 C. 6334 D. 2400 E. 2880

A6: Logarithms and indices

Question 133

The time taken (in hours) to double the initial number of bacteria is

A. 0.9 B. 1.1 C. 10 D. 11.22 E. 22.44

Question 134

The mass (in kg) of a rapidly $M(t)$ decaying radioactive substance at time t minutes after the start of an experiment is given by the equation $M(t) = 1320 \times 10^{-2t}$. After 30 seconds, the mass (in kg) is

A. 39600 B. 13200 C. 1320 D. 132 E. 132×10^{-57}

Question 135

The time (period) of swing of a pendulum (T) in seconds is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length in metres and g is 9.8. The length of the pendulum which produces a swing of period 4 seconds is nearest to

A. 0.3899 B. 12.09 C. 1.9859 D. 3.9718 E. 193.4

Question 136

If $A = 3.6 \times 10^{-5}$ and $B = 1.2 \times 10^3$ then $\sqrt{\frac{A}{B}}$ is nearest to

A. 5.73×10^2 B. 5773 C. 1.73×10^{-3} D. 1.73×10^{-4} E. 1.73×10^{-5}

Question 137

$\frac{1}{\sqrt{5}-\sqrt{3}}$ expressed with a rational denominator is equal to

A. $\sqrt{5}+\sqrt{3}$ B. $\frac{\sqrt{5}+\sqrt{3}}{2}$ C. $\frac{2}{\sqrt{5}-\sqrt{3}}$
D. $\frac{\sqrt{5}-\sqrt{3}}{2}$ E. $\sqrt{5}-\sqrt{3}$

Question 138

As one step in the procedure to solve the equation $2^{90x} + 2^{45x} - 3 = 0$, the most suitable substitution would be to use y is equal to

A. $15x$ **B.** $90x$ **C.** $45x$ **D.** 2^{90x} **E.** 2^{45x}

Question 139

Five alternative values of x are suggested below as solutions to the equation $2^x - 8x + 10 = 0$. Only one of them is reasonable. Of the following, which is it nearest to?

A. 10 **B.** 0 **C.** -2.14 **D.** 5.84 **E.** 4.85

Question 140

$2\log_{10} 2 + 3\log_{10} 5 - \log_{10} 20$ is equal to

A. $2\log_{10} 5$ **B.** $\log_{10} 109$ **C.** $\log_{10} 480$ **D.** $\log_{10}\left(\frac{19}{20}\right)$ **E.** $6\log_{10}\left(\frac{1}{2}\right)$

[Adapted from VCAA 1994 MM]

Question 141

If $3\log_{10} x - \log_{10}(x^2) = 1 + \log_{10} y$, then x is equal to

A. y **B.** $y + 10$ **C.** $\log_2 x$ **D.** $10y$ **E.** $\frac{10}{y}$

[Adapted from VCAA 1998 MM]

Question 142

If $\log_a(x^2) - 2 = 2\log_a 5$ where $a > 0$ and $x > 0$, then x is equal to

A. $5a$ **B.** $25a$ **C.** $\sqrt{10a}$ **D.** $\sqrt{(a^2 + 25)}$ **E.** $\sqrt{(25 - 2a)}$

[Adapted from VCAA 1999 MM]

Question 143

If $x = 4$ is a solution of the equation $\log_b(ax + 2) = 3$, then the exact value of a is

A. $\frac{\log_b 3 - 2}{4}$ **B.** $\frac{b}{2}$ **C.** $\frac{b^3}{4} - 2$

D. 4.5210 **E.** $\frac{b^3 - 2}{4}$

[Adapted from VCAA 2000 MM]

Question 144

a. Solve $2 \times 2^{-2x} = 2007$ for x, correct to three decimal places.

b. Simplify, by writing $2\log_a(4x + 1) - \log_a(x)$ as a single logarithm expression to base a.

[Adapted from VCAA 2002 MM]

Question 145

If $2\log_{10}(x) + 4 = 6\log_{10}(x)$ then x is equal to

A. 1 **B.** $\log_{10}(4)$ **C.** 0 **D.** $\sqrt[4]{4}$ **E.** 100

[Adapted from VCAA 2005 MM]

Solutions: A2

Question 23

'Exactly divisible by $x - 2$' means that if we substitute $x = 2$, the answer will be zero. To find the value of a:

$x^3 + 2x^2 + ax - 6$

$\therefore \quad f(2) = (2)^3 + 2(2)^2 + a(2) - 6 = 0$

$\therefore \quad 8 + 8 + 2a - 6 = 0$

$\therefore \quad 2a = -10$

$\therefore \quad a = -5$

Question 24 E

The graph of $f(x) = (x + 1)^3 - 4$ could be formed by taking the graph of $y = x^3$ (with an inflection/stationary point at the origin) and moving the origin to $(-1, -4)$. It would have point of inflection with zero gradient at that point.

Question 25 A

There is one x-intercept at 'a' (noting that it is not '$-a$', which is tricky since it is on the left; in other words, 'a' is a negative number). Thus one factor is $(x - a)$. Now, there are two other, repeated zeros (x-intercepts) at $x = b$, so that the factor $(x - b)^2$ must also be present in the answer. Thus, $f(x) = (x - a)(x - b)^2$, answer **A**.

Question 26

Let $f: R \to R$, $f(x) = 3x^2 - 12x + 18$

a. $f(x) = 3x^2 - 12x + 18$

$= 3(x^2 - 4x + 6)$ (now halve and square)

$= 3(x^2 - 4x \quad\quad + 6)$ (leave a gap)

$= 3(x^2 - 4x + 4 - 4 + 6)$ (now fill it in...)

$= 3((x^2 - 4x + 4) + 2)$

$= 3((x - 2)^2 + 2)$

$= 3(x - 2)^2 + 6$

b. The coordinates of the turning point of the graph of f are $(2, 6)$.

Question 27 B

Take a factor of x out of $x^4 + x^3 - 4x^2 - 4x$ giving $x(x^3 + x^2 - 4x - 4)$. That factor of x eliminates **D** and **E**. The others all have a factor of $(x + 1)$ so try dividing using either long division or a synthetic division method (or perhaps use graphical calculator program), we get $x(x + 1)(x^2 - 4)$, which then factorizes to give $x(x + 1)(x - 2)(x + 2)$.

Question 28

Let $P(x) = 3x^4 + 7x^2 + 7x + 11$

$\therefore \quad p(1) = 3(1)^4 + 7(1)^2 + 7(1) + 11$

$= \quad 3 + 7 + 7 + 11$

$= \quad 28$

Since this is not zero, $3x^4 + 7x^2 + 7x + 11$ is not exactly divisible by $(x - 1)$.

Question 29 A

$f(x) = 2[(x + 2)^2 - 3]$

$= \quad 2(x + 2)^2 - 6$

So the turning point will be at $(-2, -6)$.

Question 30 E

The turning point is $(-2, -4)$. That would make the equation of graph become $y = A(x + 2)^2 - 4$. Next we see that the y-intercept is at -3 or so. Substitute $x = 0$, $y = -3$ and we have

$y = A(x + 2)^2 - 4$

$\therefore \quad -3 = A(2)^2 - 4$

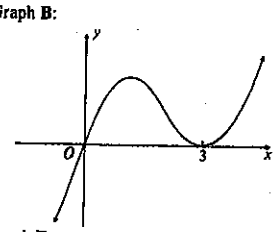
$\therefore \quad A = \frac{1}{4}$

Therefore, $A = \frac{1}{4}$ and $B = 2$ seem reasonable.

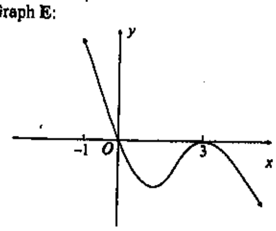
Question 31 B

For there to be a turning point at $(3, 0)$, it is necessary to have a factor of $(x - 3)^2$. That eliminates answers **A**, **C** and **D**.
You need to consider the graphs of **B** and **E**. Here, I will sketch them. Perhaps you can imagine them rather than using valuable time tapping away on your calculator...

Graph **B**:



Graph **E**:



Question 32 C

'Cubic' means three brackets like this: $f(x) = (x - \;)(x - \;)(x - \;)$
Given what we are told, the values of x where the graph touches or intersects, are either a, a, b or a, b, b. This makes the alternatives $f(x) = (x - a)(x - a)(x - b)$ or $f(x) = (x - a)(x - b)(x - b)$. Answer **C** looks good.

Question 33 D

We wish to substitute the numbers 2, -2, 0, 3 and -3 into the equation. A graphical calculator is fast (using the graph of $x^4 + 3x^3 - 4x^2 - 12x$). The results are 0, 0, 0, 90 and 0. That is, $x - 3$ is the only one which is not a factor.

Question 34 D

The graph is part of a 'positive quartic' which immediately rules out answers **C** and **E** because of the single $(\ldots -x)$ factors they have. They would make the graph become a negative quartic. The graph bounces off the x-axis at $x = 2$ so we expect a factor of $(x - 2)^2$. That eliminates **B**. Finally, $(x - 4)$ and $(x + 1)$ should appear.

Question 35 C

$f(x) = -2x^2 - 2x + 1$

$\therefore \quad f(-2) = -2(-2)^2 - 2(-2) + 1$

$= \quad -8 + 4 + 1$

$= \quad -3$

Question 36 E

$f(x) = 3x^3 - 2x - 1$

$\therefore \quad f(-1) = 3(-1)^3 - 2(-1) - 1$

$= \quad -3 + 2 - 1$

$= \quad -2$

Question 37

a. At the point of intersection,

$x^2 - 2x - 1 = 3x - 5$

$\therefore \quad x^2 - 5x + 4 = 0$

$\therefore \quad (x - 4)(x - 1) = 0$

$\therefore \quad x = 1$ or 4

The matching values of y are obtained from the equation $y = 3x - 5$. They are -2 and 7. The coordinates are $(1, -2)$ and $(4, 7)$

b. The distance is found by using $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \quad \sqrt{(4 - 1)^2 + (7 - (-2))^2}$

$= \quad \sqrt{90}$

$= \quad 3\sqrt{10}$ units.

Question 38

a. The cubic, $f(x)$, will have equation $f(x) = a(x+2)^2(x-0)$. But it also has the point (1, 18).

$\therefore \quad 18 = a(1+2)^2(1-0)$
$\therefore \quad a = 2$
$\therefore \quad f(x) = 2x(x+2)^2$.

b. $g(x)$ is found by replacing x with $(x-2)$.

$\therefore \quad g(x) = 2(x-2)((x-2)+2)^2$
$= \quad 2(x-2)(x)^2$
$= \quad 2x^2(x-2)$

Question 39 E

$x^2 + 7x + 1$
$= \quad x^2 + 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 1$
$= \quad \left(x+\frac{7}{2}\right)^2 - \frac{49}{4} + 1$
$= \quad \left(x+\frac{7}{2}\right)^2 - \frac{45}{4}$

Question 40 A

The x-value of the turning point of $y = -2x^2 - 28x - 1$ is $-\frac{b}{2a} = -\frac{-28}{-4} = -7$.
This eliminates **B**, and alternatives **D** and **E** are not 'points'. The matching y-value is

$y = -2(-7)^2 - 28(-7) - 1$
$= \quad -2(49) + 28(-7) - 1 = 97$.

The turning point is $(-7, 97)$.

Question 41 A

To have two separate solutions, $b^2 - 4ac > 0$. Thus

$(-6)^2 - 4(1)(k) > 0$
$\therefore \quad 36 - 4k > 0$
$\therefore \quad 4k < 36$
$\therefore \quad k < 9$.

Question 42 D

$(x-1-\sqrt{5})(x-1+\sqrt{5})$ is in the form of the difference of two squares. It gives

$(x-1)^2 - \sqrt{5}^2$
$= \quad x^2 - 2x + 1 - 5$
$= \quad x^2 - 2x - 4$

Question 43 B

$(x+y)^2 - (x-y)^2$
$= \quad ((x+y)+(x-y))((x+y)-(x-y))$
$= \quad (x+y+x-y)(x+y-x+y)$
$= \quad (2x)(2y)$
$= \quad 4xy$.

Question 44 B

Consider the graph without any shading first. Because we know the x-intercepts, the equation will have the form

$y = -a(x+1)(x-4)$
$= \quad -a(x^2 - 3x - 4)$

Let $a = 1$, since all the answers have $a = 1$.
$= \quad -x^2 + 3x + 4$
That means it is either **A** or **B**. Now, the origin (0, 0) is in the shaded region. Test answer A:

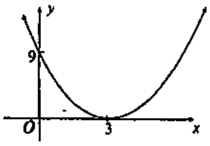
$0 \geq -0^2 + 3(0) + 4$
$\therefore \quad 0 \geq 4$ which is false.

Now test answer B:

$0 \leq -0^2 + 3(0) + 4$
$\therefore \quad 0 \leq 4$ which is true.

Question 45 A

Start by sketching the graph of $y = x^2 + 6x + 9$. This is $y = (x-3)^2$
By observation, it is only when $x = 3$ that the 'the graph will be less than or *equal* to zero'.



Question 46 B

The expansion of $(2a-b)^3$ is taken from the fact that
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ which is required knowledge. This leads to

$(2a)^3 - 3(2a)^2b + 3(2a)b^2 - b^3$
$= \quad 8a^3 - 12a^2b + 6ab^2 - b^3$

Question 47 B

If $(3-x)(2x-3)(4-3x) = 0$ then one of those brackets must be equal to zero. Thus, one of $x = 3, x = 1\frac{1}{2}$ or $x = \frac{4}{3}$ must be true.

Question 48 A

Use the remainder theorem. Substitute $x = 2$ into the equation and make the remainder zero.

$(-2)^3 - k(-2) + 2$
$\therefore \quad -8 + 2k + 2 = 0$
$\therefore \quad 2k = 6$
$\therefore \quad k = 3$

Question 49 A

Calculator techniques are obviously possible here. However, use the remainder theorem in conjunction with the hints hiding in the multiple choice answers:
Let $\quad f(x) = x^3 + 2x^2 - x - 2$
$\therefore \quad f(1) = 1^3 + 2(1)^2 - 1 - 2 = 0$
Thus, so far $(x-1)$ is a factor.

$f(-1) = (-1)^3 + 2(-1)^2 - (-1) - 2 = 0$
So $(x+1)$ is also a factor.
This makes $(x-1)(x+1)(x+2)$ the only possible answer.
Note: '2×2' grouping with $(x+2)$ taken out twice is also a possible method...

Question 50 B

The x-intercepts are at $x = -2, 1, 3$. Thus, we expect to see brackets resembling $(x+2)$, $(x-1)$ and $(x-3)$. Only alternative **B** gives all three.

Question 51 B

Here we start by looking at where the graph 'bounces off the x-axis' at $x = -2$. The other x-intercept should be at $x = 1$. Note the brackets that has $(1-x)$. It creates a negative cubic. Alternative **B** meets all of these requirements.

Question 52 B

This is essentially the graph of $y = x^3$ shifted to the right by 3 units as shifted up by 1 unit. Alternatives B and E have this. Look for the y-intercepts. Put $x = 0$ into our equation and you get $y = -26$. This best matches answer **B**.

Solutions: A3

Question 53 B

This number line section includes the −1 but not the 7. [-1, 7)

Question 54 D

These are the numbers below 6, not including 6. Thus, $(-\infty, 6)$.

Question 55 A

There are two number line sections shown to us here. They must be described to us here with two inequalities joined with the union symbol. Thus, $x < 2$ and $x \geq 3$ becomes $\{x: x < 2\} \cup \{x: x \geq 3\}$.

Question 56 C

We are looking at a number line with only one number missing from it. This means the real ser R less the number −1, or $R \setminus \{-1\}$.

Question 57 B

This is the famous set 'the real numbers plus zero'.

Question 58 A

On the number line, you could describe the numbers to the left of 3 and the numbers to the right of 3 and join them with a union. That is, $\{x: x < 3\} \cup \{x: x > 3\}$. There are other ways to describe these numbers.

Question 59 D

'Domain' means the x-values. Here, we have used the numbers from 3 to 5. Hence the answer of $\{x: 3 < x < 5\}$.

Question 60 A

'Range' means the y-values. Here we have used the numbers from −3 upwards. Thus $\{y: 2 \leq y \leq 4\}$

Question 61 E

'Domain' means the x-values. Here we have used the numbers from −2 and to the right. Thus, $\{x: x \geq -2\}$.

Question 62 D

'Domain' means the x-values. Here we have used the number line with specific numbers removed from it. The x-value −3 is certainly used but the pair of 'holes' vertically above x = 4 indicate that 4 is not being used. The answer is $R \setminus \{4\}$.

Question 63 A

The inside of that square root must not become negative. Thus, we require that

$x - 1 \geq 0$
$\therefore \quad x \geq 1$.

Question 64 E

The bottom line of a fraction must not be allowed to be equal to 0. This would happen if

$3x - 2 = 0$
$\therefore \quad x = \frac{3}{2}$

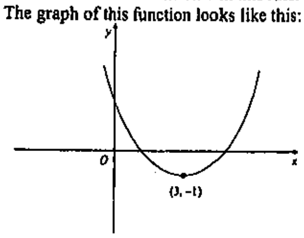
Hence, the domain is $R \setminus \{\frac{2}{3}\}$.

Question 65 A

The expression $\sqrt{x-1}$ can only be positive or zero. When you subtract 3 from it, the result will be the range from −3 upwards. Thus, $\{y: y \geq -3\}$.

Question 66 C

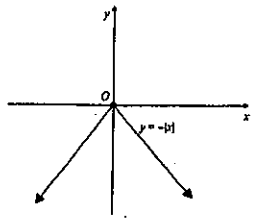
There are no restrictions on x in this rule. The graph of this function looks like this:



Note the position of the turning point, The range is $\{y: y \geq -1\}$.

Question 67 A

If you make y the subject, $y = \pm\sqrt{3-x}$. So as you construct the graph, for each x-value you choose, there will be two y-values.



Definitely, a non-function.

Question 68 E

A full circle is not a function because a vertical line can cut it twice.

Question 69 A

The graph of this function looks like this:
Thus, the range is $\{y: y \leq 0\}$.

Question 70 A

$f(x) = 2x^2 + 3$
$\therefore \quad f(-x) = 2(-x)^2 + 3$
$= \quad 2x^2 + 3$

This looks awfully like the original equation for $f(x)$. Answer A. Note that such a function is called *even*.

Question 71 A

To substitute $x = -1$ we must use the top line of the hybrid function. Thus,

$f(-1) = (-1)^2$
$= \quad 1$

Question 72 C

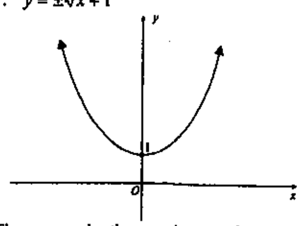
If $y = 2x - 3$ then the inverse function has the rule

$x = 2y - 3$
$\therefore \quad 2y = x + 3$
$\therefore \quad y = \frac{x}{2} + \frac{3}{2}$.

Question 73 C

If $y = x^2 - 1$ then the inverse has the rule

$x = y^2 - 1$
$\therefore \quad y^2 = x + 1$
$\therefore \quad y = \pm\sqrt{x+1}$



The answer, by the way, is not a function.

Question 74 D

Find the range of the function $y = x^2 + 1$ first. The sketch graph looks like this:
It shouldn't be necessary to actually draw it in the exam. It is a multiple choice question with only a limited amount time. The range

of the graph shown here is $\{y: y \geq 1\}$ so that the domain of the inverse is $\{x: x \geq 1\}$.

Question 75

a. Probably the best way here is to use long division:

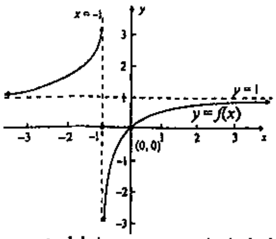
$$f(x) = \frac{x}{x+1}$$

$$\begin{array}{r} 1 \\ x+1 \overline{)\, x+0} \\ x+1 \\ -1 \end{array}$$

$\therefore \quad f(x) = 1 - \frac{1}{x+1}$

b. The only x-value that you can't have is -1 as that would make the bottom line zero. Hence $D = R \setminus \{-1\}$.

c. The graph of $f(x) = \frac{x}{x+1}$:



Be sure to label asymptotes, to include the equation and to be formal...

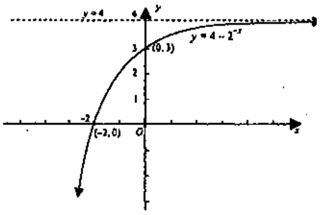
Question 76

A function f is defined by the rule $f(x) = 4 - 2^{-x}$ for $\{x: -2 \leq x \leq 6\}$.

a. $f(x) = 4 - 2^{-x}$
$\therefore \quad f(0) = 4 - 2^0$
$= \quad 4 - 1$
$= \quad 3.$
Note: the y-intercept is at $(0, 3)$

b. The x-intercepts of the graph can be found by making $f(x) = 0$.
$f(x) = 4 - 2^{-x}$
$\therefore \quad 0 = 4 - 2^{-x}$
$\therefore \quad 2^{-x} = 2^2$
$\therefore \quad -x = 2$
$\therefore \quad x = -2$
The only x-intercept is at $(-2, 0)$.

c. The graph of f:



Question 77 D

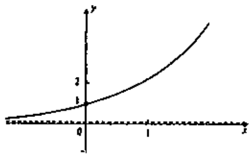
The vertical asymptote here is -1, meaning that a bracket with $(x + 1)$ is expected on the bottom line. So far, we have $y = \frac{A}{(x+1)}$. Now, when $x = 0$, $y = 1$ (from the graph given to us), thus

$1 = \frac{A}{(0+1)}$
$\therefore \quad A = 1$
$\therefore \quad y = \frac{1}{(x+1)}$

Thus, answer **D**.

Question 78 D

The graph of $f(x) = 2^x$ looks like this:



By inspecting this graph, it can be seen that yes, it has range R^+, yes it has domain R and yes it has a horizontal asymptote at $y = 0$. But when $x = 0$, $y = 1$. And, oh yes, the gradient is always positive...

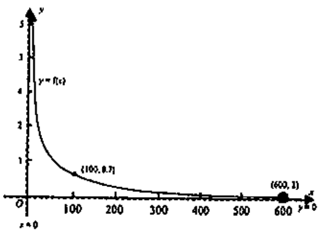
Question 79 E

Because of that $x - 4$ on the bottom line, $y = \frac{2}{x-4} + 3$ can't have $x = 4$. Thus, we have a vertical asymptote there. Also, it is like the $1/x$ graph shifted up 3 units, so the horizontal asymptote is at $y = 3$.

Question 80

Be sure to answer each of the questions in each part.

a. Note the endpoint and the asymptotes.



b. You must solve simultaneously the equations $y = 2$ and $y = 2.5 - 0.9\log_{10}(x)$. Either draw the two graphs on the same calculator screen and find the intersection, or use 'solve' on your calculator. I got $(3.59, 2)$. But it asks for the answer to the nearest pixel so the answer is 4 pixels.

Solutions: A4

Question 81 B

The answer is obtained mainly by looking carefully at the asymptotes.
Because the graph asymptotes to the positive x-axis, the inverse graph will asymptote to the positive y-axis. The other 'original' asymptote was $x = 1$ so the new graph will go through $y = 1$.

Question 82 B

If we start with the reflection, $y = \sqrt{x}$ becomes $y = -\sqrt{x}$. The graph is then translated 2 units to the right so it becomes $y = -\sqrt{x-2}$. Finally, the translation '1 unit down' gives us $y = -\sqrt{x-2} - 1$.

Question 83 C

Clearly, the $f(x)$ graph needs to be turned 'upside down' or reflected over the x-axis. Then the result needs to be stretched vertically away from the x-axis by a factor of 2. That gives alternative **C**.

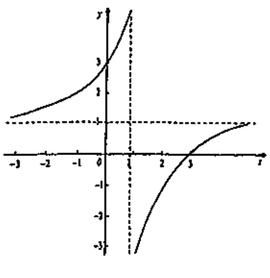
Question 84

We start with $y = \frac{1}{x}$. Now, if it is dilated from the y-axis by a factor of 2, then the equation will be multiplied by 2. So far, we have $y = 2 \times \frac{1}{x}$. Next, we reflect the graph across the y-axis, by making x become $-x$. Now the equation is $y = 2\frac{1}{(-x)} = -\frac{2}{x}$.
Next, we shift right by 1 unit, making x become $(x - 1)$, giving $y = -\frac{2}{x-1}$.

Finally, a translation of +1 unit parallel to the y-axis simply adds 1 to the whole equation.

a. $y = -\frac{2}{x-1} + 1$

b. The domain is $R \setminus \{1\}$ and range is $R \setminus \{1\}$. This sketch may help you determine the domain and range).



Question 85 E

Step 1. To reflect in the y-axis, replace x with $-x$.
Step 2. For a dilation of 3 units from the x-axis, multiply the equation by 3.

Therefore, make x become $-x$. Since x appears only once on the equation, our answer so far will be $y = \sqrt{-x}$. Now put a 3 in front of the whole thing for that dilation *from* the x-axis, giving $y = 3\sqrt{-x}$.

Question 86

a. The equation $f(x) = x^2$ will become $g(x) = (x + 4)^2$.

b. The equation becomes $h(x) = (x + 4)^2 + 3$.

c. The equation becomes $k(x) = (2x + 4)^2 + 3$.

Note that the dilation is parallel to the x-axis, away from the y-axis and it is applied after part b. That is a little tricky!

Question 87 E

$y = x^3$ becomes $y = (x + 1)^3$, which then becomes $y = (3x + 1)^3$. Note that the expression 'from the y-axis' means that it is a horizontal dilation, and that the '$\frac{1}{3}$' will becomes a '2'. Be sure not to take the 3 out as a common factor unless it is cubed. Definitely, answer **D** is wrong.

Question 88 E

If x is replaced by $3x$, the graph is 'squashed horizontally' by a factor of 3 towards the y-axis and stretched vertically away from the x-axis by a factor of 5. Stated more formally, a dilation from the x-axis by a scale factor of 5 and a dilation from the y-axis by a scale factor of $\frac{1}{3}$.

Question 89 C

If $y = f(x)$ has a vertical asymptote $x = 3$, then f^{-1} will have a horizontal asymptote at $y = 3$.

Question 90

The graph of the function with rule $y = x^2$ is transformed and becomes the graph with rule $y = 3(x - 2)^2$.

Do the '3' first then the '2' afterwards. The graph of $y = x^2$ is dilated by a factor of $\frac{1}{3}$ parallel to the x-axis from the y-axis. It is then translated by 2 units parallel to the x-axis.

Solutions: A5

Question 91 E

Start with the amplitude of the graph. Clearly, it is 2, the vertical distance from the symmetrical centre to the highest point on the graph. Thus, $a = 2$.
Next, the graph can be seen to have a period of π, the distance between two matching points on the graph. That means the value of b is found from the formula

$\pi = \frac{2\pi}{b}$
$\therefore \quad b = 2$

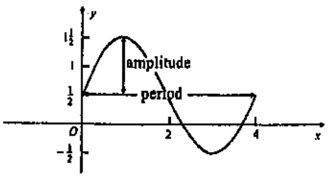
Answer **E** has such a graph shifted right by $\frac{\pi}{9}$ making it the best answer.

Question 92 B

If $f: R \to R$, $f(x) = 2\cos(3x) - 1$ then the amplitude is 2. Thus, we are restricted to answers **A**, **B** and **C** so far.
The period is $\frac{2\pi}{3}$ allowing answer **B**. The range will be from $2 - 1 = 1$ to $-2 - 1 = -3$ confirming answer **B**.

Question 93

This might help:



a. The period = 4 units

b. The amplitude = 1 unit (NOT 1.5 units).

Question 94

a. The period is 4 units.
b. The amplitude of the graph is 2 units. Now, the number that goes in front of the x is $\frac{2\pi}{4} = \frac{\pi}{2}$. If it has been shifted up by 2 units, the equation becomes

$y = 2\sin\left(\frac{\pi x}{2}\right) + 2$

Question 95

a. The height of the low tide is $6 - 2 = 4$ m.
b. The height of the high tide is $6 + 2 = 8$ m.
c. The period of the graph with equation $h(t) = 6 - 2\sin\left(\frac{4\pi t}{25}\right)$ is $\frac{2\pi}{n}$

$= \quad 2\pi \div \frac{4\pi}{25}$

$= \quad 2\pi \times \frac{25}{4\pi} = \frac{25}{2}$ hours.

That time will take us from a low tide to the next low tide. That is, $12\frac{1}{2}$ hours. Since the low tide was at midnight, the next low tide will be at 30 minutes past midday.

Question 96

a. The maximum height above the ground will be when the value of $6\sin\left(\frac{\pi t}{30}\right)$ is as big as possible. That is, 6 metres. Thus, the maximum height will be 21 metres.

b. When $t = 0$, $x(0) = 15 + 6\sin(0)$
$= \quad 15$ metres.

c. To have a height of 15 metres, it must be true that $6\sin\left(\frac{\pi t}{30}\right) = 0$

Solutions: A6

Question 112 A

$\log_a b = c$
$\therefore\ a^c = b$

Question 113 B

If $\log_5 125 = 3$
$\therefore\ 5^3 = 125$

Question 114 A

Let $\log_3 81 = x$
$\therefore\ 3^x = 81$
$\therefore\ 3^x = 3^4$
$\therefore\ x = 4$

Question 115 C

The part following the word log (the argument) must be positive.
$x - 3 > 0$
$\therefore\ x > 3$

Question 116 D

The part following the word log (the argument) must be positive.
For $y = \log_{10}(5 - 2x)$,
$5 - 2x > 0$
$\therefore\ -2x > -5$
$\therefore\ x < \frac{5}{2}$

Question 117 B

$\log_a 3 + \log_a 2$
$= \log_a 6$

Question 118 E

$3\log_x 5 = \log_x 5^3 = \log_x 125$

Question 119 C

$\log_3 5 + \log_3 9 - \log_3 15$
$= \log_3 \frac{5 \times 9}{15}$
$= \log_3 3$
$= 1$

Question 120 E

$\log_e (x^3 y^2)$
$= \log_e (x^3) + \log_e (y^2)$
$= 3\log_e x + 2\log_e y$

Question 121 A

$\log_3 6$
$= \log_3 (3 \times 2)$
$= \log_3 (3) + \log_3 (2)$

Question 122 A

$\log_3 \sqrt{3}$
$= \log_3 3^{\frac{1}{2}}$
$= \frac{1}{2}\log_3 3$
$= \frac{1}{2} \times 1$
$= \frac{1}{2}$

Question 123 D

$\frac{1}{3}\log_2 64$
$= \frac{1}{3}\log_2 (2^6)$
$= \frac{1}{3} \times 6\log_2 (2)$
$= \frac{1}{3} \times 6 \times 1$
$= 2$

Question 124 D

$\frac{\log_{10} 125}{\log_{10} 25}$
$= \frac{\log_{10}(5^3)}{\log_{10}(5^2)}$
$= \frac{3\log_{10}(5)}{2\log_{10}(5)}$
$= \frac{3}{2} = 1\frac{1}{2}$

Question 125 C

$x \log_{10} 3 = \frac{3}{2}\log_{10} 9$
$\therefore\ x \log_{10} 3 = \frac{3}{2}\log_{10} 3^2$
$\therefore\ x \log_{10} 3 = 3\log_{10} 3$
By comparing the two sides,
$x = 3$

Question 126 E

$\log_{10} x - \log_{10}(x - 3) = \log_{10} 4$
$\therefore\ \log_{10} \frac{x}{x-3} = \log_{10} 4$
$\therefore\ \frac{x}{x-3} = 4$
$\therefore\ 4x - 12 = x$
$\therefore\ 3x = 12$
$\therefore\ x = 4.$

Question 127 B

$2\log_{10} 4 - 2\log_{10} 5 + \log_{10} 50$
$= \log_{10} 4^2 - \log_{10} 5^2 + \log_{10} 50$
$= \log_{10} \frac{16 \times 50}{25}$
$= \log_{10} 32$
$= \log_{10} 2^5$
$= 5\log_{10} 2$

Question 128 C

$2^x = 5$
$\therefore\ \log_{10}(2^x) = \log_{10}(5)$
$\therefore\ x\log_{10}(2) = \log_{10}(5)$
$\therefore\ x = \frac{\log_{10}(5)}{\log_{10}(2)}$
≈ 2.32

Question 129 C

The graph of $y = \log_{10}(x - 1)$ will have a vertical asymptote at $x = 1$.

Question 130 D

Consider $\left(\frac{1}{2}\right)^x = 4$ first.
$\therefore\ \left(\frac{1}{2^x}\right) = 2^2$
$\therefore\ 2^{-x} = 2^2$
$\therefore\ -x = 2$
$\therefore\ x = -2$
If we want If $\left(\frac{1}{2}\right)^x > 4$ then making x a smaller number (more to the left on the number line) will achieve that. Thus,
$x < -2$
If you are unsure of that last step, try raising $\frac{1}{2}$ to various powers.

Question 131 A

If $N = 1200\,(2)^{0.1t}$ then when $t = 0$ days, N will be 1200 bacteria.

Question 132 A

$N = 1200 \times (2)^{0.1 \times 2}$
$= 1378.43$
≈ 1378 bacteria.
(1379 bacteria would be ok. What to do with 0.43 of a bacteria is not well defined. You would need to check under a microscope).

Question 133 C

The initial number of bacteria was 1200. Now we want 2400.
$N = 1200 \times (2)^{0.1t}$
$\therefore\ 2400 = 1200 \times (2)^{0.1t}$
$\therefore\ 2 = (2)^{0.1t}$
$\therefore\ 1 = 0.1t$
$\therefore\ t = 10$ days.

Question 134 C

$M(t) = 1320 \times 10^{-2t}$. Let $t = 0.5$ minutes (watch those units),
$\therefore\ M = 1320 \times 10^{-1}$
$= 132$ kg.

Question 135 D

$T = 2\pi\sqrt{\frac{l}{g}}$
$\therefore\ 4 = 2\pi\sqrt{\frac{l}{9.8}}$
$\therefore\ 16 = 4\pi^2 \times \frac{l}{9.8}$
$\therefore\ l = \frac{16}{4\pi^2} \times 9.8$
≈ 3.9718

Question 136 D

If $A = 3.6 \times 10^{-5}$ and $B = 12 \times 10^3$ then
$\sqrt{\frac{A}{B}}$
$= \sqrt{\frac{3.6 \times 10^{-5}}{12 \times 10^3}}$
$= \sqrt{3 \times 10^{-8}}$
$\approx 1.73 \times 10^{-4}$

Question 137 B

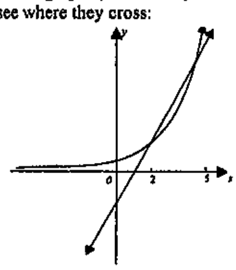
$\frac{1}{\sqrt{5} - \sqrt{3}}$
$= \frac{1}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$
$= \frac{\sqrt{5} + \sqrt{3}}{5 - 3}$
$= \frac{\sqrt{5} + \sqrt{3}}{2}$

Question 138 E

$2^{90x} + 2^{45x} - 3 = 0$
$= a^2 + a - 3 = 0$
If you were to let $a = 2^{45x}$.

Question 139 E

$2^x - 8x + 10 = 0$
$\therefore\ 2^x = 8x - 10$
Sketch the graphs $y = 2^x$ and $y = 8x - 10$ and see where they cross:



One of them looks to be nearly $x = 4.85$.

Question 140 A

$2\log_{10} 2 + 3\log_{10} 5 - \log_{10} 20$
$= \log_{10} 2^2 + \log_{10} 5^3 - \log_{10} 20$
$= \log_{10} \frac{4 \times 5^3}{20}$
$= \log_{10} \frac{5^3}{5}$
$= \log_{10} 5^2$ or $2\log_{10} 5$

Solutions: A6

Question 141 D

$3\log_{10} x - \log_{10}(x^2) = 1 + \log_{10} y$
$\therefore\ \log_{10} x^3 - \log_{10}(x^2) - \log_{10} y = 1$
$\therefore\ \log_{10} \frac{x^3}{x^2 y} = 1$
$\therefore\ \log_{10} \frac{x}{y} = 1$
$\therefore\ \frac{x}{y} = 10$
$\therefore\ x = 10y$

Question 142 A

$\log_a (x^2) - 2 = 2\log_a 5$
$\therefore\ \log_a (x^2) - \log_a 5^2 = 2$
$\therefore\ \log_a \left(\frac{x^2}{5^2}\right) = 2$
$\therefore\ x^2 = 25a^2$
$\therefore\ x = 5a$ noting that $x > 0$.

Question 143 E

$\log_b (ax + 2) = 3$
$\therefore\ \log_b (4a + 2) = 3$
$\therefore\ b^3 = 4a + 2$
$\therefore\ b^3 - 2 = 4a$
$\therefore\ a = \frac{b^3 - 2}{4}$

Question 144

a. One way is to sketch two graphs on the same axis: $y = 2 \times 2^{-2x}$ and $y = 2007$. You will need a calculator window of $x = -10$ to $x = 10$ and $y = -100$ to $y = 3000$. They intersect at $x = -4.9854$ or approximately $x = -4.985$.

b. $2\log_e (4x + 1) - \log_e (x)$
$= \log_e (4x + 1)^2 - \log_e (x)$
$= \log_e \frac{(4x + 1)^2}{x}$

Question 145 C

$2\log_{10}(x) + 4 = 6\log_{10}(x)$
$4 = 6\log_{10}(x) - 2\log_{10}(x)$
$\therefore\ 4 = 4\log_{10}(x)$
$\therefore\ \log_{10}(x) = 1$
$\therefore\ x = 10$
It is not a bad idea to check that the answer is ok by substituting it back into the original equation. Here, nothing negative or zero results after either occurrences of the word log when we do that.